Report on the paper

Dunford-Pettis operators on the space of Bochner integrable functions

by

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The topic of the paper is little old fashioned and rather far from the main stream of current functional analysis. The author refers to an old result of J. Bourgain characterizing operators $T : L^1[0,1] \to E$ by the following way: T is Dunford-Pettis if and only if the composition of T with the injection $L^p[0,1] \hookrightarrow L^1[0,1]$ is compact for some $p \in (1, \infty]$. The main theorems included in the paper extend Bourgain's characterization of Dunford-Pettis operators onto the case of Bochner integrable functions $L^1(X)$ and L^p spaces are replaced by vector valued Orlicz spaces $L^\Phi(X)$ generated by Young functions Φ. The first main theorem (Theorem 2.1), which shows that an operator $T : L^1(X) \to Y$ with compact restriction to $L^\Phi(X)$ is Dunford-Pettis, does not require any assumptions about Banach spaces X and Y. The proof of Theorem 2.1 is standard. Similarly the proof of the second main result (Theorem 2.3) consists on combinations of several earlier published results by a few authors. Theorem 2.3 is proved under a restrictive assumption that X is reflexive (many properties of such spaces can be easily obtained by almost verbatim repeating of proofs concerning appropriate facts concerning the L^1 space of real valued functions). The author does not make any remark if the reflexivity can be weakened.

On the other hand the paper is fully correct, the results are new and original and they complete our knowledge about Dunford-Pettis operators forming very important class of operators between Banach spaces. Moreover the results should be interesting for a wide group of readers. Therefore I recommend the paper, but without a great enthusiasm, for publication in the Proceedings of the Jozef Marcinkiewicz Centenary Conference.

Below I enclose a list of remarks which should be considered by the author preparing the final version of his paper.

page 1, the first line of the abstract, it should be: and let X be a real Banach space.

page 1, the fifth line of the Introduction and preliminaries, it should be: restricted to L^p (for some $p \in (1, \infty]$) is compact.

page 1, line 5 form below the condition $\lim_{t\to\infty} \frac{\Phi(t)}{t} > 0$ is superfluous for non zero function Φ. This fact is an easy consequence of previous assumptions because $\frac{\Phi(t)}{t}$ is nondecreasing: $0 < s < t$ implies $\Phi(s) = \Phi(\frac{s}{t}t + (1 - \frac{s}{t})0) \leqslant \frac{s}{t}\Phi(t)$, and so the limit exists. Moreover, if $\Phi(t_0) > 0$ then $\frac{\Phi(t_0)}{t_0} \leqslant \frac{\Phi(t)}{t}$ for all $t \geqslant t_0$, i.e., the limit is strictly positive.

page 2, line 10 from above, it should be: From now we assume (or From now we will assume).

page 2, line 11 from above, it should be: and X^*, Y^* denote their Banach duals.

page 2, line 15 from above, it should be: together with the norm ... is a Banach space and it is usually called...

page 2, line 22 from above: the Δ_2-condition (for large arguments) means exactly that $\limsup_{t\to\infty} \frac{\Phi(2t)}{\Phi(t)} < \infty$. The condition $\lim_{t\to\infty} \frac{\Phi(2t)}{\Phi(t)} < \infty$ is formally stronger; the author should explain why he defines the condition Δ_2 is such way.

page 3, line 15 from above, it should be: To show that $T(H)$ is relatively compact in $(Y, \|\cdot\|_Y)$ it is enough to show, in view of [D, p.5], that for ...

page 3, line 19 from above: the theorem from [DU, p. 101] is not appropriate (the second part of Theorem 4 from [DU, p.104] is adequate),

page 6, line 3 from below: I do not know why the word Weakly is written in brackets.